Projectile motion

Projectile

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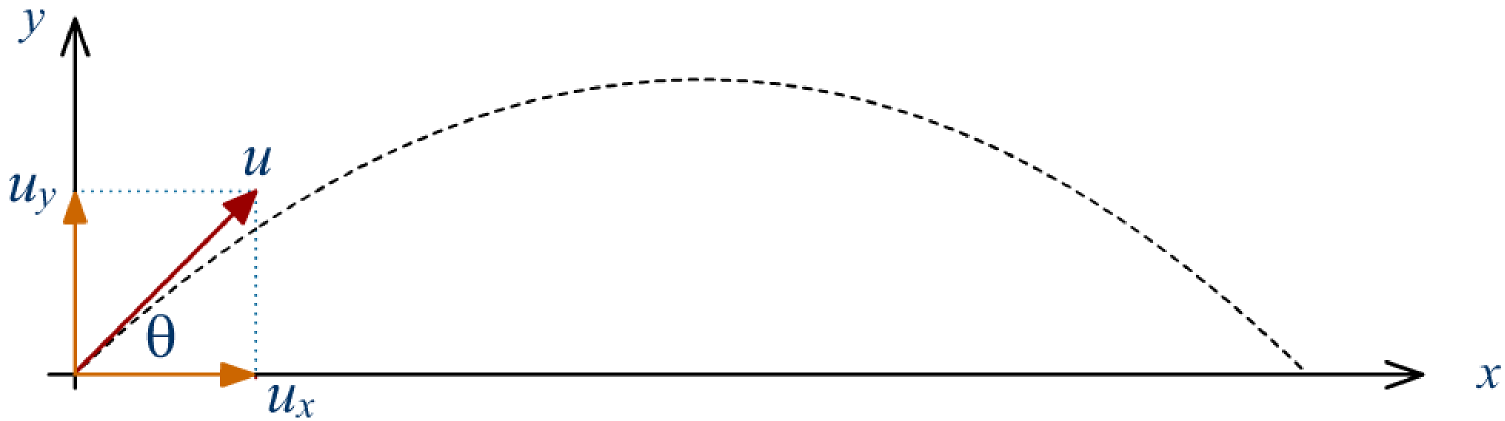

A body is projected from ground with initial velocity u at an angle θ w.r.t. the ground is called a projectile.

Initial velocity of the body may be resolved into two mutually perpendicular components.

$$u_x = u\cos(\theta)$$ Horizontal component of initial velocity

$$u_y = u\sin(\theta)$$ Vertical component of initial velocity

Horizontal component of velocity remains constant. Vertical component of velocity decreases during ascent, becomes zero momentarily at the highest point and increases in the downward direction during descent (because of g) .

Projectile : Maximum height (H)

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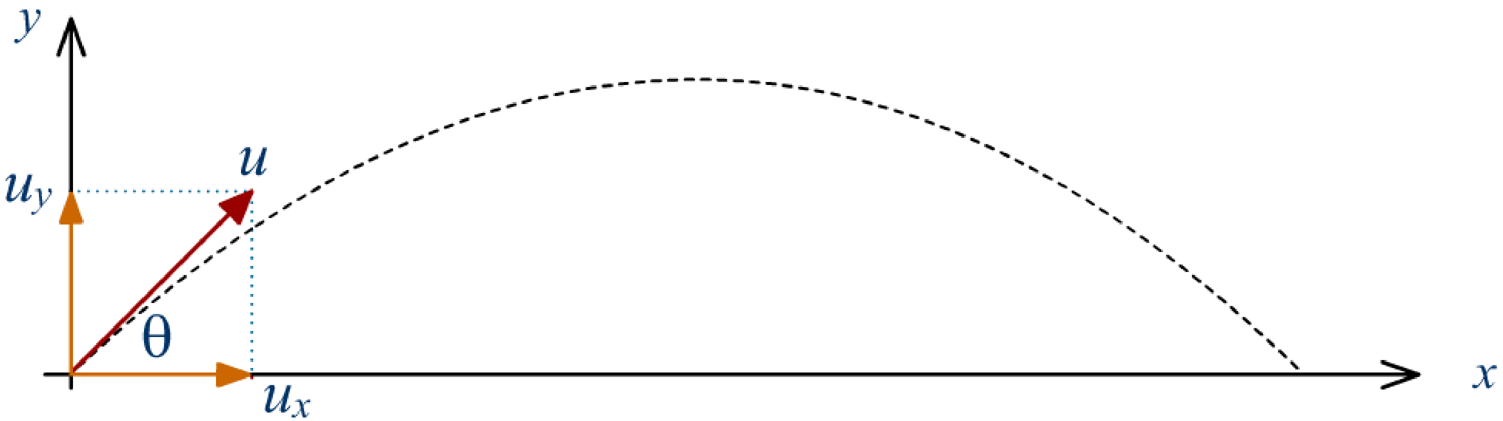

It is the height attained by the body (from the point of projection) at which vertical component of its velocity becomes zero.

$$v^2 - u^2 = 2aS$$

$$v_y^{\ 2} - u_y^{\ 2} = 2a_y S_y$$

Acceleration is (g) and at the highest point v_y becomes zero therefore

$$0 - u^2 \sin^2(\theta) = -2gH$$

$$2gH = u^2 \sin^2(\theta)$$

$$\boxed{H = \frac{u^2 \sin^2(\theta)}{2g}} \quad 1$$

Note : For θ = 90º, the formula reduces to that of a vertically projected body

Projectile : time of ascent (TOA)

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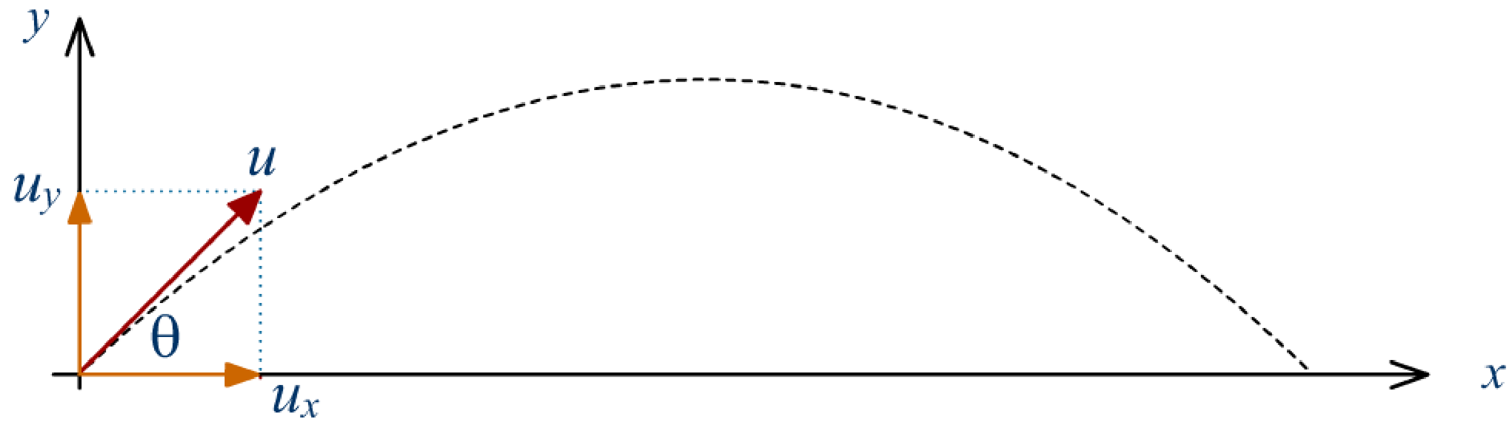

Time of ascent is the time interval (from the instant of projection) to the instant at which the vertical component of the projectile becomes zero.

$$v = u + at$$

$$v_y = u_y + a_y t$$

Acceleration is (g) and at the highest point v_y becomes zero

$$0 = u\sin(\theta) - gt$$

$$t = \frac{u\sin(\theta)}{g}$$

$$\boxed{TOA = \frac{u\sin(\theta)}{g}} \quad (2)$$

Note : For θ = 90°, the formula reduces to that of a vertically projected body

Projectile : time of flight (TOF)

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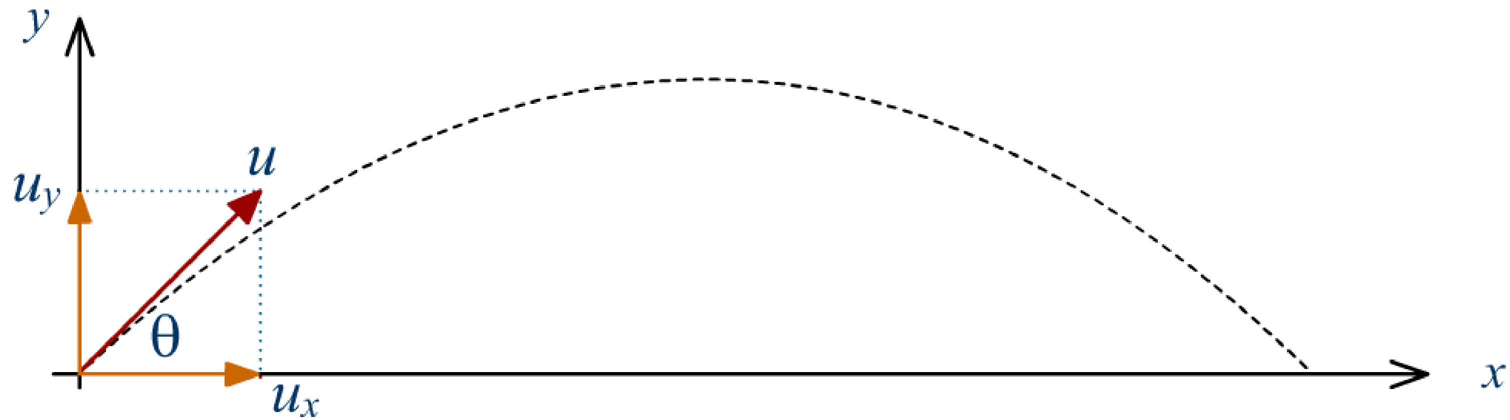

Time of flight is the time spent by the body in flight as it reaches the ground

$$TOF = TOA + TOD$$

As ascent is symmetric to descent, the time taken by it for ascent is equal to the time taken by it for descent. Therefore

$$TOF = 2 \times TOA$$

$$\boxed{TOF = \frac{2u\sin(\theta)}{g}} \quad (3)$$

Projectile : Horizontal range (R)

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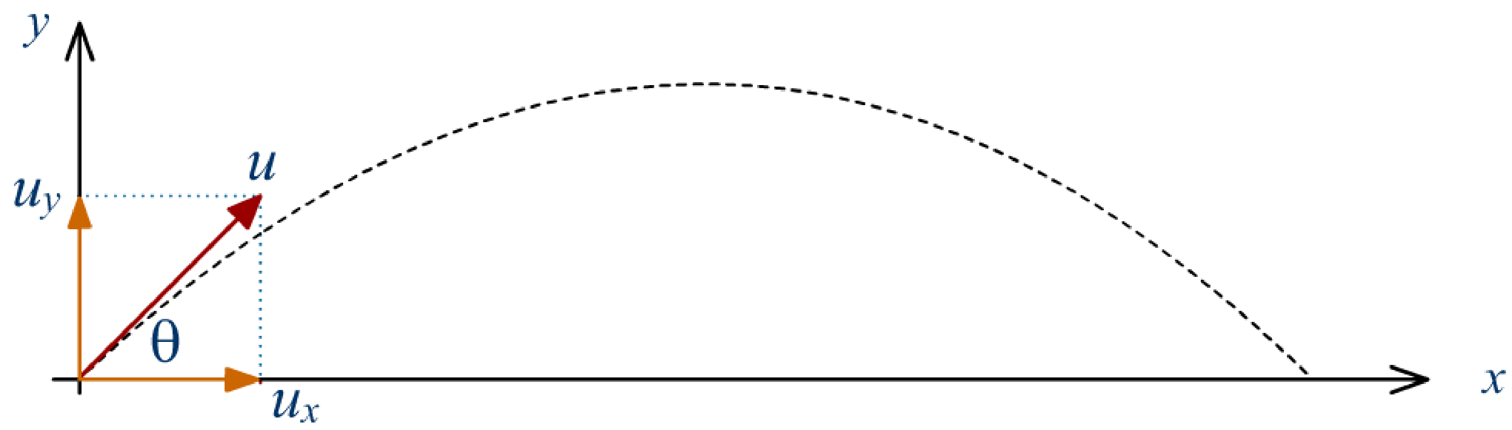

It is the horizontal displacement of the projectile as it reaches the ground.

$$S = ut + \frac{1}{2}at^2$$

$$S_x = u_x t + \frac{1}{2}a_x t^2$$

There is no acceleration in the horizontal direction, therefore

$$S_x = u\cos(\theta)\times t$$

Using t = TOF we get

$$R = u\cos(\theta)\frac{2u\sin(\theta)}{g}$$

Using sin(2θ) = 2 sin(θ) cos(θ)

$$\boxed{R = \frac{u^2\sin(2\theta)}{g}} \quad (4)$$

Projectile : Path of a projectile (*parabolic path*)

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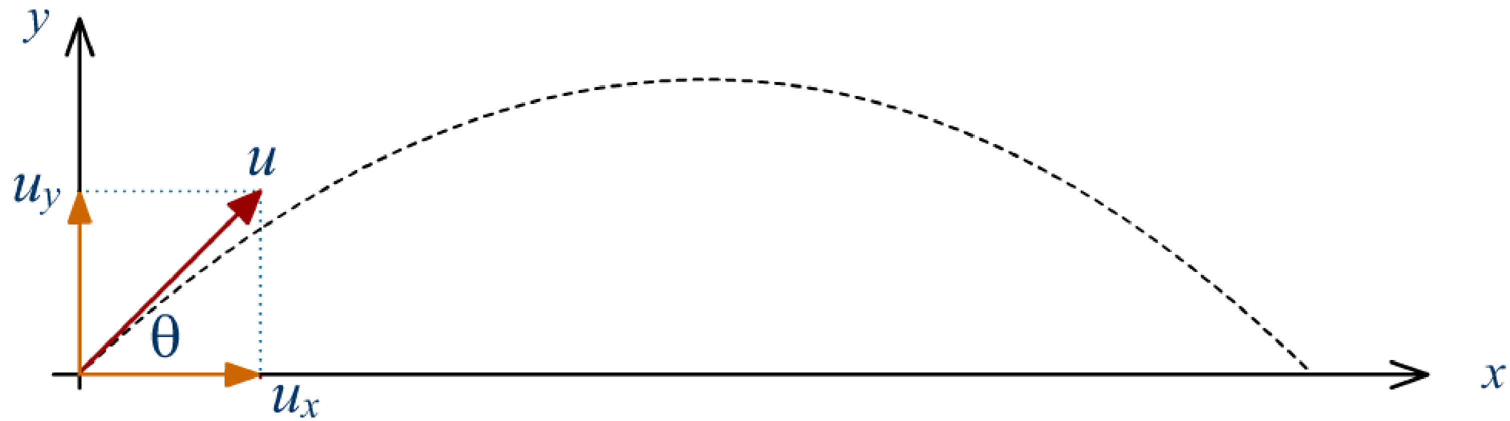

Considering horizontal component of motion we get

$$x = u\cos(\theta)\times t$$

$$\Rightarrow \quad t = \frac{x}{u\cos(\theta)} \quad \text{—— i}$$

Considering vertical component of motion we get

$$y = u\sin(\theta)\times t - \frac{1}{2}g t^2 \quad \text{—— ii}$$

Substituting eq(i) in eq(ii) we get

$$y = u\sin(\theta)\frac{x}{u\cos(\theta)} - \frac{1}{2}g\left(\frac{x}{u\cos(\theta)}\right)^2$$

$$\boxed{y = x\tan(\theta) - \frac{gx^2}{2u^2\cos^2(\theta)}} \quad \text{—— 4}$$

$$y = Ax - Bx^2$$

This is an equation of parabola

Motion in a plane

Projectile :

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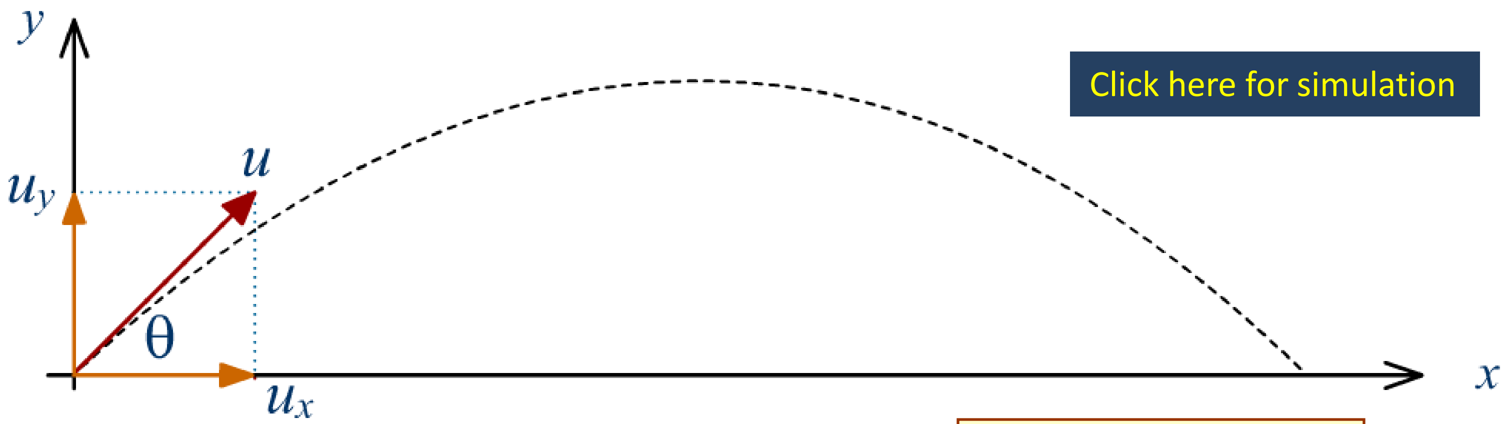


$u_x = u\cos(\theta)$ Horizontal component of u

$v_x = u\cos(\theta)$ constant

$u_y = u\sin(\theta)$ Vertical component of u

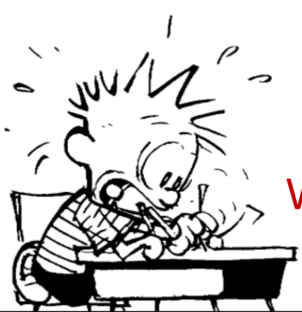
$v_y = u\sin(\theta) - gt$ Changes with time

$$H = \frac{u^2\sin^2(\theta)}{2g}$$

$$TOA = \frac{u\sin(\theta)}{g}$$

$$R = \frac{u^2\sin(2\theta)}{g}$$

$$y = x\tan(\theta) - \frac{gx^2}{2u^2\cos^2(\theta)}$$



Before using any equation...
Which body ?
Which component of motion?
Starting & ending points